Technical Notes

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Energy Absorption Heating and Ignition of Energetic Solids

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Nomenclature

K_a	=	absorption coefficient, 1/cm
k	=	thermal conductivity, $\mathrm{W/(cm \cdot K)}$
q	=	absorbed radiant flux, $\mathrm{W/cm^2}$
T	=	temperature, K
T_0	=	initial temperature, K
T_s	=	ignition temperature, K
t	=	time, s
t_s	=	ignition time, s
X	=	dimensionless coordinate normal to surface, positive into a sold, $x/(1/K_a)$
x	=	coordinate normal to surface, positive into solid [cm]
α	=	thermal diffusivity, $\mathrm{cm^2/s}$
β	=	dimensionless temperature, $(T-T_0)/(q/kK_a)$
β_s	=	in-depth absorption parameter or dimensionless ignition temperature, $(T_s-T_0)/(q/kK_a)$
τ	=	dimensionless time, $t\alpha K_a^2$
τ_s	=	dimensionless ignition time, $t_s\alpha K_a^2$

Introduction

IN A recent paper, the influence of surface absorption and in-depth absorption on ignition time was analyzed by Brewster [1] based on an analytical model with volumetric exponential absorption in energetic solids. A new parameter was defined to evaluate the relative importance of surface absorption and in-depth absorption in affecting the ignition time of solids. However, the governing equation in the paper was solved by a numerical technique, which offered limited information in understanding the physical phenomena. The objective of this Note is to give an analytical solution of the governing equation, and the importance of the new parameter in affecting the ignition time will be further demonstrated.

Analytical Solutions of the Governing Equation

As proposed by Brewster [1], a model problem of energetic solid radiant heating and ignition is governed by the following equation:

$$\frac{1}{\alpha}\frac{\partial T}{\partial t}=\frac{\partial^2 T}{\partial x^2}+\frac{qK_a}{k}\exp(-K_a x) \tag{1}$$

with the boundary conditions and initial condition as

$$T(x,t=0)=T_0,\qquad \left.\frac{\partial T}{\partial x}\right|_{x=0}=0,\quad \text{and}\quad T(x\to\infty,t)=T_0 \tag{2}$$

This equation is solved numerically in the previous work [1]. Fortunately, this equation has an exact solution, as follows [2]:

$$\begin{aligned}T(x,t)=&\frac{2q}{k}\sqrt{\alpha t}\,iErfc\left(\frac{x}{2\sqrt{\alpha t}}\right)-\frac{q}{kK_a}\exp(-K_a x)\\&+\frac{q}{2kK_a}\exp\left(\alpha K_a^2 t-K_a x\right)Erfc\left(K_a\sqrt{\alpha t}-\frac{x}{2\sqrt{\alpha t}}\right)\\&+\frac{q}{2kK_a}\exp\left(\alpha K_a^2 t+K_a x\right)Erfc\left(K_a\sqrt{\alpha t}+\frac{x}{2\sqrt{\alpha t}}\right)+T_0\end{aligned} \tag{3}$$

where $Erfc(x)$ is the complementary error function and $iErfc(x)$ is the integration of the complementary error function with

$$iErfc(x)=\frac{1}{\sqrt{\pi}}e^{-x^2}-xErfc(x)$$

Results and Discussion

Based on the analytical solution of Eq. (1) and the associated boundary conditions and initial condition of Eq. (2), the surface temperature reads

$$T(0,t)=\frac{2q}{k}\sqrt{\frac{\alpha t}{\pi}}-\frac{q}{kK_a}+\frac{q}{kK_a}\exp\left(\alpha K_a^2 t\right)Erfc(K_a\sqrt{\alpha t})+T_0 \tag{4}$$

By using the parameter-free variables [1] $\beta=(T-T_0)/(\frac{q}{kK_a})$, $X=x/(1/K_a)$, and $\tau=t\alpha K_a^2$, the parameter-free solution becomes

$$\begin{aligned}\beta(X,\tau)=&2\sqrt{\tau}\,iErfc\left(\frac{X}{2\sqrt{\tau}}\right)-\exp(-X)\\&+\frac{1}{2}\exp(\tau-X)Erfc\left(\sqrt{\tau}-\frac{X}{2\sqrt{\tau}}\right)\\&+\frac{1}{2}\exp(\tau+X)Erfc\left(\sqrt{\tau}+\frac{X}{2\sqrt{\tau}}\right)\end{aligned} \tag{5}$$

The nondimensional surface temperature is

$$\beta(0,\tau)=2\sqrt{\frac{\tau}{\pi}}-1+\exp(\tau)Erfc(\sqrt{\tau}) \tag{6}$$

With the given boundary conditions, the maximum temperature occurs at the wall, and the ignition temperature is first reached at the surface. The time τ_s to reach the ignition temperature is determined by solving the following:

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$$\beta_s = 2\sqrt{\frac{\tau_s}{\pi}} - 1 + \exp(\tau_s)Erfc(\sqrt{\tau_s}) \tag{7}$$

where $\beta_s = (T_s - T_0)/(q/kK_a)$ is the nondimensional ignition temperature, also called the in-depth absorption parameter, and β_s is monotonously increasing with increasing τ_s. In other words, τ_s also monotonously increases with increasing β_s. To study the relationship of the ignition time to β_s, some limiting cases will be discussed. When τ_s is small, Eq. (7) can be approximated by

$$\beta_s(0, \tau_s) = \tau_s + \text{HOT} \tag{8}$$

by using a Taylor expansion around zero with

$$e^x = 1 + x + \frac{x^2}{2} + \text{HOT}$$

and

$$Erfc(x) = 1 - \frac{2}{\sqrt{\pi}}x + \frac{2}{3\sqrt{\pi}}x^3 + \text{HOT}$$

where HOT denotes high-order terms. For very large τ_s, the following is obtained:

$$\beta_s(0, \tau_s) = 2\sqrt{\frac{\tau_s}{\pi}} - 1 + \frac{1}{\sqrt{\pi}}\left(\frac{1}{\sqrt{\tau_s}}\right) - \frac{1}{2\sqrt{\pi}}\left(\frac{1}{\sqrt{\tau_s}}\right)^3 + \frac{3}{4\sqrt{\pi}}\left(\frac{1}{\sqrt{\tau_s}}\right)^5 + \mathrm{O}\left(\frac{1}{\sqrt{\tau_s}}\right)^7 \tag{9}$$

by applying the asymptotic expansion series of the $Erfc$ function at infinity

$$Erfc(x) = e^{-x^2}\left[\frac{1}{\sqrt{\pi}}\left(\frac{1}{x}\right) - \frac{1}{2\sqrt{\pi}}\left(\frac{1}{x}\right)^3 + \frac{3}{4\sqrt{\pi}}\left(\frac{1}{x}\right)^5 + \mathcal{O}\left(\frac{1}{x}\right)^7\right]$$

A plot of Eq. (7) showing the time to reach ignition temperature and the limiting cases is shown in Fig. 1. As seen in the plot, the limiting cases are approached when τ_s is very small (i.e., $\tau_s < 0.01$) or very large (i.e., $\tau_s > 100$). Because the ignition temperature, the absorption coefficient, and the thermal conductivity are material properties assumed to be constant in this analysis, the definition of β_s implies $\beta_s \propto 1/q$ if other parameters remain constants. Using the definition of β_s and Eq. (7), the relation between the ignition time and the absorbed radiant flux is

$$q = \frac{(T_s - T_0)kK_a}{2\sqrt{\tau_s/\pi} - 1 + \exp(\tau_s)Erfc(\sqrt{\tau_s})} \tag{10}$$

Equation (10) gives the heat flux necessary to reach a given ignition temperature T_s in a desired ignition time t_s or τ_s. From Eqs. (8) and (9) and the definition of β_s, when the limiting conditions for surface absorption ($K_a \to \infty$, $\beta_s \gg 1$) and uniform volumetric absorption ($K_a \to 0$, $\beta_s \ll 1$) are approached, the ignition time becomes proportional to $1/q^2$ and $1/q$, respectively.

An interesting observation is that when $\beta_s = 1$, solving Eq. (7) yields $\tau_s = 2.206$ [i.e., the ignition time $t_s = 2.206/(\alpha K_a^2)$], which is only dependent on material properties. However, for the surface absorption model with $\beta_s = 1$, the ignition time is $t_s = \pi/(4\alpha K_a^2)$, and for the uniform volumetric absorption model, $t_s = 1/(\alpha K_a^2)$. Both the surface absorption model and the uniform volumetric model significantly underpredict the ignition time for a given absorption coefficient. For moderate values of β_s, there is no exact power-law relationship of the ignition time to the absorbed radiant flux. However, a power-law fit can be found for a moderate range of τ_s. A power-law fit was obtained for $1 \le \tau_s \le 10$ and the comparison is illustrated in Fig. 2. The power-law fit for the given range of τ_s based on the least-squares method is

$$\beta_s = 0.5804\tau_s^{0.6813} \tag{11}$$

which is equivalent to the following:

$$t_s \propto q^{-1.4678} \tag{12}$$

The preceding obtained power-law exponent is closer to some experimental results [3]. This shows that the in-depth absorption model could provide more reasonable prediction for certain moderate values of β_s compared with the surface absorption model and the uniform volumetric absorption model. The in-depth absorption could have a significant effect on the estimate of ignition time under different absorbed radiant fluxes. To determine the power-law exponent for different β_s values, manipulating Eq. (7) yields

$$\frac{\mathrm{d}[\ell n(\beta_s)]}{\mathrm{d}[\ell n(\tau_s)]} = \frac{\tau_s Erfc(\sqrt{\tau_s})}{2\exp(-\tau_s)\sqrt{\tau_s/\pi} - \exp(-\tau_s) + Erfc(\sqrt{\tau_s})} \tag{13}$$

Equation (13) is plotted in Fig. 3 and shows that the power-law exponent changes from one for small β_s to 0.5 for large β_s. With the definition of β_s, it is obvious that in terms of the ignition time and the absorbed radiant flux, the power-law exponent changes from -2 to -1 for different β_s. Based on Eq. (13), is found that the following equation holds:

$$\beta_s \propto \tau_s^n \tag{14}$$

where $0.5 < n < 1$ and n depends on τ_s. If we keep all the parameters constant except K_a, with the definitions of and β_s and τ_s, we obtain

$$t_s \propto K_a^{1-2n} \tag{15}$$

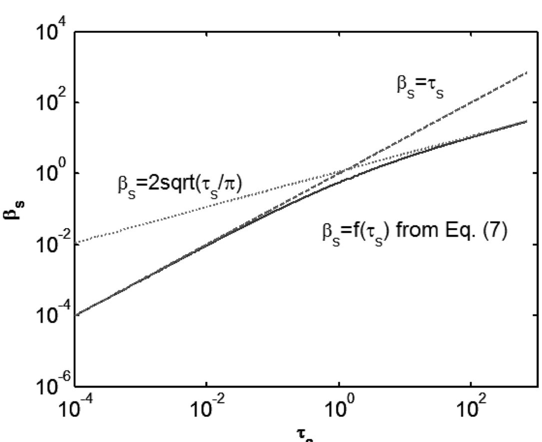

Fig. 1 The relationship between β_s and τ_s showing the time to reach ignition temperature for different models.

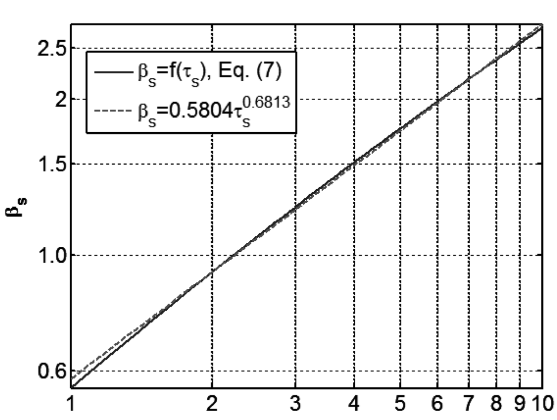

Fig. 2 The relationship between β_s and τ_s for Eq. (7) and a power-law fit ($1 \le \tau_s \le 10$).

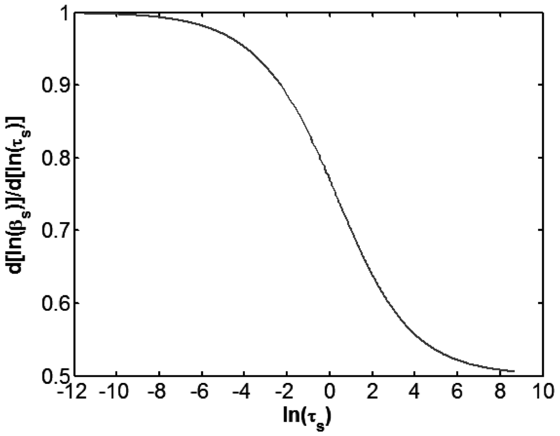

Fig. 3 The relationship of $d[\ln(\beta_s)]/d[\ln(\tau_s)]$ and $\ln(\tau_s)$ showing the power-law exponent change with varying τ_s.

Because we know that $0.5 < n < 1$, Eq. (15) indicates that if all other parameters/variables are fixed and only K_a varies, ignition time decreases monotonically with increasing K_a.

Conclusions

An exact solution of the governing heat conduction equation of an analytical model for volumetric absorption heating and ignition of energetic solids was presented. The exact solution provides more flexibility than the numerical solution in understanding the energy absorption and ignition mechanism of energetic solids. The importance of the new parameter β_s defined by Brewster [1] was further illustrated. The results provide accurate relationship of the ignition time to the absorbed radiant flux for different values of β_s. The surface absorption model and uniform volumetric model are only some special cases of the current study.

References

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